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AN IMPROVED METHOD FOR OPTIMUM DESIGN OF MECHANICALLY AND THERMALLY LOADED STRUCTURES

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16. Abstract

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An improved algorithm for resizing of structures subjected to thermal stresses is presented. In this algorithm the mechanical portions of the stresses are driven to their maximum allowable values. The thermal stresses are used to adjust the allowable values of the mechanical stresses. The new algorithm is exercised for a number of truss structures of varying complexity and compared with ordinary fully stressed design (i.e., resizing based on total stresses). It was found that for a wing truss structure with significant thermal loading, the new algorithm converged significantly faster than the ordinary fully stressed design algorithm.

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SUMMARY

The problem of obtaining the minimum-mass design of mechanically and thermally loaded structures is the subject of this report. The special nature of thermal stresses with regard to their response to resizing of structural members is discussed. It is shown that conventional resizing procedures which are based on driving the total stress to its allowable value may be inefficient when the thermal stress in an element makes up a significant fraction of the total stress.

An improved algorithm for resizing of structures subjected to thermal stresses is presented. In this algorithm the mechanical portions of the stresses are driven to their maximum allowable values. The thermal stresses are used to adjust the allowable values of the mechanical stresses. The new algorithm is exercised for a number of truss structures of varying complexity and compared with ordinary fully stressed design (i.e., resizing based on total stresses). It was found that for a wing truss structure with significant thermal loading, the new algorithm converged significantly faster than the ordinary fully stressed design algorithm.

INTRODUCTION

Procedures have been developed in recent years to automate the preliminary design of structures subjected to a variety of load situations and under various constraints dictated by the service environment of the structure. The mathematical programing approach (ref. 1) is generally applicable to problems which can be formulated in terms of an objective function (such as structural mass) and an appropriate set of constraints expressed in terms of the design variables. This method has been used to optimize the design of structures under constraints on stress and displacements (ref. 2), vibration frequency (ref. 3), and flutter speed (ref. 4).

Another basic approach to structural optimization is the use of optimality criteria. The most widely used optimality criterion is fully stressed design (FSD). Applications of this approach are described in numerous publications (refs. 5, 6, and 7, for example).

This paper deals with the problem of calculating the optimum (minimum-mass) design of mechanically and thermally loaded structures with an emphasis on those problems in which the thermal loads are either comparable to the mechanical loads or dominate the mechanical loads. The usual manner of treating this class of problems is to combine algebraically the mechanical and thermal loads and to compute the stresses due to the combined loads. These total stresses are then used as the basis for resizing the structure en route to the optimum design by either math programing techniques or fully stressed design techniques. The objection to using the total stresses in fully stressed design is that this approach does not take account of the fact that the thermal stresses in a structural element are less sensitive to resizing than are the mechanical stresses. If the thermal stress in a member is a significant fraction of the total stress, fully stressed design will perform needless iterations in attempts to reduce the thermal stresses. Moreover, if the thermal stresses are sufficiently large, no amount of resizing alone will yield an acceptable design.

An improved procedure[1] for the design of structures under combined thermal-mechanical loading is described herein. Specifically, the procedure monitors the thermal stresses and mechanical stresses individually. The effective allowable stress for a given element is the algebraic difference between the material allowable stress and the thermal stress in that element. The structure is resized in such a way as to drive the mechanical stresses to the effective allowable stresses. In this way, the new procedure tends to avoid the shortcomings of total-stress-based resizing procedures when thermal stress is a significant fraction of the combined stress. In order to treat problems for which the thermal stresses are so large that no acceptable design can be obtained by structural resizing alone, a modified algorithm has also been developed. This algorithm helps to identify how much the thermal stress needs to be reduced by means other than resizing in order that an acceptable design can be found. A design is obtained which corresponds to the reduced thermal stresses. The design, as well as the amount of thermal-stress reduction, is of use provided that some means of thermal-stress alleviation such as insulation, heat sinks, or constraint release is also incorporated by the designer. Calculations are carried out for several sample problems including a heated wing structure with a moderate number of design variables.

SYMBOLS

Values are presented in both SI and U.S. Customary Units. The calculations were made in U.S. Customary Units.

[1]A preliminary and simplified version of this procedure was suggested informally to the authors by Dr. I. U. Ojalvo of the Grumman Aerospace Company.

A	design variable, area of a truss member
E	Young's modulus
$[G]$	stress-strain matrix
h	height of three-bar truss (see fig. 1)
$[K]$	stiffness matrix
$\{L\}$	load vector
P	mechanical load
$[S]$	stress-displacement matrix
T	temperature
T_o	stress-free temperature
$\{u\}$	displacement vector
α	coefficient of linear thermal expansion
β	thermal-stress reduction factor
ρ	density
δA	change in design variable
σ	stress in a structural element
σ_a	allowable stress

Subscripts:

i	iteration number
M	mechanical

T thermal

x,y,z direction along X-, Y-, and Z-axis, respectively

ILLUSTRATION OF THE BASIC PROBLEM

In order to illustrate the basic differences in the behavior of mechanical and thermal stresses, consider the simple but illuminating example of a 45° planar three-bar truss shown in figure 1. The loading consists of a load with components P_x and P_y in the x- and y-direction, respectively, as well as element temperature changes T_1, T_2, and T_3 above some reference temperature T_o. If $A_1 = A_3$, the displacements u and v in the x- and y-direction, respectively, and the stresses σ_1, σ_2, and σ_3 in the bars are given by:

$$\left.\begin{aligned}
u &= \frac{\sqrt{2}P_x h}{A_1 E} + \alpha h(T_1 - T_3) \\
v &= \frac{P_y h}{E\left(\frac{\sqrt{2}}{2}A_1 + A_2\right)} + \frac{\alpha(T_1 + T_3)\left(\frac{A_1}{A_2}\right) + \sqrt{2}\alpha T_2}{\sqrt{2} + \left(\frac{A_1}{A_2}\right)} h \\
\sigma_1 &= \frac{P_x}{\sqrt{2}A_1} + \frac{P_y}{\sqrt{2}A_1 + 2A_2} + \frac{E\alpha(T_2 - T_1 - T_3)}{2 + \sqrt{2}\left(\frac{A_1}{A_2}\right)} \\
\sigma_2 &= \frac{P_y}{\frac{\sqrt{2}}{2}A_1 + A_2} - \frac{E\alpha(T_2 - T_1 - T_3)\left(\frac{A_1}{A_2}\right)}{\sqrt{2} + \left(\frac{A_1}{A_2}\right)} \\
\sigma_3 &= \frac{P_y}{\sqrt{2}A_1 + 2A_2} - \frac{P_x}{\sqrt{2}A_1} + \frac{E\alpha(T_2 - T_1 - T_3)}{2 + \sqrt{2}\left(\frac{A_1}{A_2}\right)}
\end{aligned}\right\} \qquad (1)$$

It is instructive to examine the behavior of the stresses with respect to area changes. The stresses caused by mechanical loads can be altered by changing either A_1 or A_2. In contrast, the thermal stresses are functions of area ratios. If the thermal loads are sufficiently large, no amount of resizing will give a satisfactory design.

Recognition of the differing dependence of mechanical and thermal stresses on member size just discussed is not incorporated into existing automated design procedures; rather, the basis for resizing a structural member has traditionally been the total stress (i.e., the algebraic sum of the stresses due to mechanical loads and thermal loads). The procedure developed in this paper monitors the mechanical and thermal stresses separately and exploits the basic differences in their response to member resizing.

ANALYSIS AND RESIZING PROCEDURES

Calculations of displacements and stresses for the structures are based on finite element methods. The appropriate equations are

Equilibrium equation:

$$[K]\{u\} = \{L\} \tag{2}$$

Constitutive equation:

$$[S]\{u\} - [G]\{\alpha\}(T - T_o) = \{\sigma\} \tag{3}$$

Fully Stressed Design (FSD)

In the conventional fully stressed design algorithm, the structural members are resized according to the ratio of the total stress to the allowable stress in the following manner:

$$A_{i+1} = \frac{\sigma_i}{\sigma_a} A_i \tag{4}$$

where

i	iteration number
A	design variable (membrane thickness, bar area, etc.)
σ_a	allowable stress in the member having the same sign as σ_i
σ_i	total stress in the member at the ith iteration

For the linear problems considered herein, $\sigma_i = \sigma_{M,i} + \sigma_{T,i}$ where

$\sigma_{M,i}$ stress due to mechanical loads (mechanical stress)

$\sigma_{T,i}$ stress due to thermal loads (thermal stress)

The procedure continues until, in at least one loading condition, each member is fully stressed in the sense that the total stress equals the allowable stress or a size constraint is encountered. Thus FSD drives each member toward the condition

$$\frac{\sigma_M + \sigma_T}{\sigma_a} = 1 \tag{5}$$

FSD With Taylor Series Reanalysis

In FSD with Taylor series reanalysis, complete reanalyses involving new decompositions of the stiffness matrix are performed only during selected iterations rather than during each iteration (ref. 8). In previous applications of this procedure the load vector has been independent of the design variables. In the present paper, the extension of the procedure for loads proportional to design variables is indicated. The algorithm proceeds as:

$$\left\{u_{i+1}\right\} = \left\{u_i\right\} + \left[\frac{\partial u}{\partial A}\right]_i \left\{\delta A_i\right\} \tag{6}$$

$$\left\{\sigma_{i+1}\right\} = [S]\left\{u_{i+1}\right\} - [G]\left\{\alpha\right\}(T - T_o) \tag{7}$$

$$\left\{\delta A_i\right\} = \left\{A_i\left(\frac{\sigma_i}{\sigma_a} - 1\right)\right\} \tag{8}$$

The matrix $\left[\frac{\partial u}{\partial A}\right]$ is computed by differentiating equation (2) with respect to A and rearranging to obtain:

$$[K]\left[\frac{\partial u}{\partial A}\right] = \left[\frac{\partial L}{\partial A}\right] - \left[\frac{\partial K}{\partial A}\right]u \tag{9}$$

The matrix $\left[\frac{\partial L}{\partial A}\right]$ is null when only mechanical loads are present. But for thermal loading the matrix contains terms proportional to the structural temperatures. This method will be referred to as Taylor series FSD. One point to be noted regarding Taylor series FSD is the need to start reasonably close to the final design, otherwise the approach will not converge. Since the final design cannot be discerned at the beginning of the design process, the method used in this paper to generate a starting point for Taylor series FSD is to execute a number of ordinary FSD iterations before entering into the Taylor series FSD procedure.

Thermal Fully Stressed Design (TFSD)

In the FSD methods previously discussed, the basis for structural resizing has been the total stresses and is predicated on maintaining a constant force in each member during an iteration. The basis for the new algorithm (TFSD) is that during each iteration the thermal stress and the mechanical force each remain constant. For a change in member size from the ith to the (i + 1)th iteration the algorithm is

$$A_{i+1} = \left(\frac{\sigma_{M,i}}{\sigma_{a,M} - \sigma_{T,i}}\right) A_i \tag{10}$$

where $\sigma_{a,M}$ is the allowable stress having the sign of σ_M. Equation (10) is appropriate provided the quantity in parentheses is positive. The algorithm drives each member toward the condition

$$\frac{\sigma_M}{\sigma_{a,M} - \sigma_T} = 1 \tag{11}$$

Thus, the mechanical stress is driven toward an allowable value that is adjusted by the amount of the thermal stresses.

ILLUSTRATIVE CALCULATIONS AND DISCUSSION

A number of calculations are presented for the design of truss-type structures. These structures are convenient because of their obvious simplicity, ease of programing solutions, and the availability of results in the open literature. It is emphasized that the methods used in this paper are in no way restricted in applicability to truss structures. The results and comparisons in this section are intended to illustrate the effect of using the TFSD algorithm in comparison to FSD.

Heated Three-Bar Truss

A heated three-bar truss was considered. This multiple-load case example was chosen for the initial set of calculations because it is a problem solved in the literature for combined thermal-mechanical loading. The truss geometry and material properties together with the three loading conditions are shown in figure 1. Results for this problem for a multiple-load calculation based on the combination of load cases 1 to 3 were published in reference 7. Present results are obtained by FSD, Taylor series FSD, and TFSD. Two iterations were required for convergence to within 5 percent of the final mass given in table 1 for each method. The results presented in table 1 show that the present designs are in precise agreement with the FSD result of reference 7 and give a slightly heavier design than the math programing result in reference 7. The results in this table and in subsequent sections of the paper are considered to be fully converged.

Four-Bar Pyramid

A four-bar pyramid is shown in figure 2 along with the four single-loading cases. The first two cases have only mechanical loads, and results corresponding to these loads are published in references 2 and 9. Two additional load cases involving thermal loads are defined. These consist of temperature loads on each bar along with a concentrated load at the apex of the pyramid. The load is upward in case 3 and downward in case 4. There are no published check results for thermal loading of the pyramid; thus, a general purpose optimization code based on mathematical programing was used to generate such check results. The program, denoted AESOP (ref. 10), was also used to obtain results for cases 1 and 2.

The results of the calculations are summarized in table 2. There was no difference between FSD and Taylor series FSD for any of the calculations; thus, there is no separate column in table 2 for Taylor series FSD. Further, for the remaining designs all the methods were in excellent agreement except that the math programing procedure in AESOP gave a slightly lighter design in case 1. The FSD, Taylor series FSD, and TFSD algorithms all converged to within 5 percent of the final mass in table 2 in two iterations.

Heated 25-Bar Transmission Tower

The heated 25-bar transmission tower described in figure 3 and table 3 is subjected to mechanical loads in addition to thermal loads as indicated. No solutions have been published for this structure under thermal loading.

Results for this problem are presented in table 4. Inspection of this table indicates that the FSD, Taylor series FSD, and TFSD give nearly identical results and the AESOP result gives a slightly lighter design. In this example, the temperatures were selected so that the thermal stresses would be much lower than the mechanical stresses. This was done to evaluate the performance of TFSD relative to FSD and Taylor series FSD for such a problem. All three procedures converged to within 5 percent of the final mass in two iterations. Thus, for predominantly mechanically loaded structures, there seems to be no difference between the efficiency of TFSD and FSD or Taylor series FSD.

Evidently, the Taylor series approximate reanalysis technique is not particularly advantageous when used in a fully stressed design context. The reason is that the initial FSD steps necessary to assure convergence bring the design so close to the final design that only a few more iterations are required to achieve a converged design for engineering purposes. The advantages of the Taylor series method become more pronounced as larger numbers of reanalysis steps are required.

A 136-Bar Wing Truss

The 136-bar wing truss shown in figure 4 was chosen to illustrate the TFSD procedure on a moderate-sized structure (in terms of the number of design variables) with significant thermal loading. The structure was modeled after the hypersonic wing of reference 11 and is subjected to a three-dimensional temperature distribution similar to that in reference 11. In addition, a uniform pressure load and concentrated elevon moment act on the wing. The element connections for the wing are tabulated in table 5 and the grid-point locations are in table 6. The loads and temperatures are listed in table 7. The resulting design from FSD is given in table 8. The TFSD result is given in table 9 and is essentially the same as the FSD result. The significant result of this calculation is the number of iterations required to converge to within 5 percent of the final weight. Only 4 iterations were required by TFSD for this degree of convergence and FSD required 16 iterations. The reason for the faster convergence of TFSD can be illustrated by looking at a typical element which is predominantly thermally stressed. For example, bar 42 had, in the initial design, a thermal-stress ratio σ_T/σ_a of 0.78 and a mechanical stress ratio σ_M/σ_a which was negligibly small. As a result, ordinary FSD, which resizes on the basis of the total stress ratio, reduced the bar area only by 22 percent on the first iteration. However, TFSD immediately reduced the area of bar 42 to minimum gage on the first iteration. The FSD procedure did not arrive at a minimum-gage area for bar 42 until after 27 iterations. Thus, the TFSD procedure gives a faster converging scheme for design of structures with significant thermal-stress levels.

MODIFIED PROCEDURE FOR EXCESSIVELY THERMALLY STRESSED STRUCTURES

In dealing with the design of structures under mechanical and thermal loading, it is possible to encounter problems in which, because of the nature of thermal stresses discussed previously, no acceptable design can be found by resizing alone. Specifically, suppose that the thermal stress in a member exceeds the allowable stress and the thermal stress in that member cannot be decreased by structural resizing. If, in that same member, the mechanical and thermal stresses have the same sign, it is clear that neither equation (5) nor equation (11) can be satisfied. In this situation, the algorithm of equation (10) as well as the FSD algorithm (eq. (4)) will be unable to converge.

One remedy is to make a suitable reduction in the thermal stress by means other than resizing. For example, allowing relative motion between adjacent structural members or releasing boundary constraints could cause a significant reduction in thermal stresses (ref. 12). Another method is to reduce the structural temperatures by means of thermal control procedures such as adding insulation or active and passive heat sinks.

Assume that, by one or more of the above procedures, the thermal stresses in appropriate members are reduced to a fraction β of their values, thus permitting an acceptable design to be reached by resizing. Ideally β would be mathematically related to:

(1) Parameters which characterize thermal control procedures such as insulation thickness, magnitude of heat sinks, etc.

(2) A weight or other penalty associated with thermal control

(3) The required temperature reduction or the amount of boundary constraint release

Such characterizations of β do not presently exist and are beyond the scope of this work which makes only a first step toward dealing with thermal-mechanical design problems which heretofore were not treatable. In the spirit of making such a first step, consider a modified TFSD algorithm which is the same as TFSD except that $\sigma_{T,i}$ is replaced by $\beta_i \sigma_{T,i}$. The magnitude of β_i will be interpreted as the fraction of thermal stress which can be accommodated by the design at the ith iteration. A simplifying assumption will be made; namely, that β_i is the same for all elements and will be selected on the basis of the "worst" element. As a result, some element thermal stresses are reduced more than is necessary. A different approach, which has not been implemented, would be to compute individual values of β_i for each element and reduce the thermal stress in each element by the required fraction.

The modified TFSD algorithm has the form

$$A_{i+1} = \frac{\sigma_{M,i}}{\sigma_{a,M} - \beta_i \sigma_{T,i}} A_i \tag{12}$$

Equation (12) will be referred to as modified TFSD. The appropriate value of β_i is obtained by examining all members whose mechanical and thermal stresses have the same sign and identifying that member having the largest thermal stress. Let that member be designated "member N." The value of β_i depends on the magnitude of the thermal and mechanical stresses in that member, and is chosen so that in member N

$$\sigma_{M,i} + \beta_i \sigma_{T,i} = \sigma_{a,M} \tag{13}$$

Three cases are examined. First, if member N is not thermally overstressed, that is,

$$\left(\frac{\sigma_{T,i}}{\sigma_{a,M}}\right)_{\text{member N}} < 1 \tag{14}$$

no thermal-stress reduction is necessary ($\beta_i = 1$) and the algorithm is the same as TFSD (eq. 10). Second, if member N is both thermally and mechanically overstressed, that is,

$$\left(\frac{\sigma_{T,i}}{\sigma_{a,M}}\right)_{\text{member N}} > 1 \tag{15}$$

and

$$\left(\frac{\sigma_{M,i}}{\sigma_{a,M}}\right)_{\text{member N}} > 1 \tag{16}$$

β is calculated from

$$\beta_i = \left(\frac{\frac{1}{2}\,\sigma_{a,M}}{\sigma_{T,i}}\right)_{\text{member N}} \tag{17}$$

The choice of the factor 1/2 in equation (17) is arbitrary as any value between zero and unity would be satisfactory. The rationale for doing this is that the above choice of β_i, when substituted into the resizing formula (eq. (12)), results in an improved design at the next iteration. Third, if member N is thermally overstressed but not mechanically overstressed, that is,

$$\left(\frac{\sigma_{T,i}}{\sigma_{a,M}}\right)_{\text{member N}} > 1 \tag{18}$$

but

$$\left(\frac{\sigma_{M,i}}{\sigma_{a,M}}\right)_{\text{member N}} < 1 \tag{19}$$

β_i is calculated so as to satisfy equation (13) for member N. Thus,

$$\beta_i = \left(\frac{\sigma_{a,M} - \sigma_{M,i}}{\sigma_{T,i}}\right)_{\text{member N}} \tag{20}$$

Before illustrating the modified TFSD algorithm, it is suggested that the basic ideas in this section could also be adapted for use in a mathematical programing approach. It would be necessary to replace the usual stress constraint which is

$$\sigma - \sigma_a \leqq 0 \tag{21}$$

by a new constraint

$$\sigma_M - \beta\sigma_T\sigma_{a,M} \leqq 0 \tag{22}$$

where β is calculated by equation (17) or (20).

ILLUSTRATION OF THE MODIFIED ALGORITHM

The modified TFSD procedure is illustrated by applying it to the 136-bar wing truss subjected to the same mechanical loads as in the previous calculation (load case 1) but having temperatures which are 2.5 times as large as those previously applied. The details of this loading condition (load case 2) are given in table 10. When the FSD procedure was attempted for this problem, it failed to converge because no acceptable design exists for the full temperatures. When the modified TFSD algorithm was applied, it converged with a value of β of 0.426. In this example, member N was bar 47 which had a thermal-stress ratio $\sigma_T/\sigma_{a,M}$ of 1.984 and a mechanical stress ratio $\sigma_M/\sigma_{a,M}$ of 0.155. The design is given in table 11.

The above design information is, of course, only of value if the designer has means at his disposal for controlling the thermal stresses to some extent. If appropriate thermal-stress reductions cannot be made, the designer is at least alerted to the possible need for a configurational change or release of a rigid boundary restraint. The point is that the modified TFSD does give information relative to the need for reduced temperatures and gives the corresponding design based on such reductions. This type of information is not ordinarily available from other methods.

The fact that β converges to a value less than unity is not always a reliable indicator of the need for thermal-stress reduction. In other words, it is a necessary, but not sufficient condition, for failure of TFSD and FSD to converge for the full thermal stresses. This situation is discussed more fully in the appendix.

CONCLUDING REMARKS

This report deals with the problem of obtaining the minimum-mass design of structures under combined thermal-mechanical loading. An improved resizing algorithm is described which addresses the relative insensitivity to changes in member size of thermal stresses compared to mechanical stresses. Specifically, the procedure monitors the thermal stresses and mechanical stresses individually. The effective allowable stress for a given element is the algebraic difference between the material allowable stress and thermal stress in that element. The structure is resized in such a way as to drive the mechanical stresses to the effective allowable stresses.

The new algorithm (called thermal fully stressed design or TFSD) is compared to the usual fully stressed design (FSD) procedure for some truss-type structures treated in the literature for predominantly mechanical loads to confirm the validity and accuracy of the TFSD procedure. Additional comparisons are made with an FSD algorithm in which approximate reanalysis of the structure is performed using a Taylor series approximation. The procedure is extended in this paper to be applicable to thermal loads. Results from all of the methods were in excellent agreement and converged in only a few iterations. In order to determine the effectiveness of TFSD for a moderate-sized problem, a wing structure with 136 design variables was designed by both TFSD and FSD. For this structure, the thermal loads dominated the mechanical loads and it was found that although both procedures converged to the same result, TFSD required far fewer iterations to converge than did FSD.

In order to treat problems for which the thermal stresses are so large that no acceptable design can be obtained by structural resizing alone, a modified TFSD algorithm has been developed. This procedure is similar to TFSD and helps to identify how much the thermal stress need be reduced by means other than resizing in order for an acceptable design to be found. A design is obtained which corresponds to the reduced thermal stresses. The design, as well as the amount of thermal-stress reduction, is useful provided that some means of thermal-stress alleviation such as insulation, heat sinks, or constraint release are also incorporated by the designer.

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APPENDIX

INSUFFICIENCY OF $\beta < 1$ FOR INDICATING THE NEED FOR THERMAL-STRESS REDUCTION

When β in the modified TFSD algorithm (eqs. (12) to (20)) converges to a value less than unity, it is a necessary, but not sufficient, condition for failure of TFSD and FSD to converge without thermal-stress reduction.

The reason for the insufficiency is that the converged value of β depends on the thermal stress in "member N." Thus, if the thermal-stress ratio in that member exceeds unity, $\beta < 1$, and if the ratio is less than unity, $\beta = 1$. There is a marginal situation when, because of the slightly different resizing formulas, modified TFSD arrives at a design having a thermal-stress ratio in member N slightly exceeding unity, but FSD arrives at a design with a value slightly less than unity. In this situation, modified TFSD will converge to a value of $\beta < 1$ while FSD is able to converge for the full temperatures.

This marginal situation will be illustrated by another wing-truss example. In this example, the mechanical loads are the same as in case 1 but the temperatures are 25 percent higher. This loading condition (case 3) is given in table 12. When modified TFSD was applied to this problem, the design given in table 13 was obtained. In this design, member N was bar 47 which had a thermal-stress ratio of 1.075 and a mechanical stress ratio of 0.230 which resulted in a β-value of 0.715. At the same time, when FSD was applied to this example, a converged solution was obtained for the full temperatures. This FSD result is shown in table 14.

It should be observed in this example, that the marginal nature of the problem is associated with the value of the thermal-stress ratio and not necessarily with the value of β. Because of the definition of β and the manner of its calculation (eqs. (13) to (20)), the thermal-stress ratio in member N may be only slightly larger than unity, but the corresponding value of β may be significantly smaller than unity. It is therefore necessary to check carefully the thermal stresses in designs corresponding to $\beta < 1$ and by use of engineering judgment determine if thermal-stress reduction is needed.

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TABLE 1.- RESULTS FOR DESIGN OF A HEATED THREE-BAR TRUSS

Areas of Bars

Bar	Present results						Reference 7			
	FSD		Taylor series FSD		TFSD		FSD		Math program	
	cm^2	in^2	cm^2	in^2	cm^2	in^2	cm^2	in^2	cm^2	in^2
1	7.813	1.211	7.813	1.211	7.813	1.211	7.813	1.211	7.200	1.116
2	2.006	.311	2.006	.311	2.006	.311	2.006	.311	3.574	.554
3	10.76	1.668	10.76	1.668	10.76	1.668	10.76	1.668	9.968	1.545

Total Mass

Present results						Reference 7			
FSD		Taylor series FSD		TFSD		FSD		Math program	
kg	lbm	kg	lbm	kg	lbm	kg	lbm	kg	lbm
4.016	8.854	4.016	8.854	4.016	8.854	4.016	8.854	3.955	8.720

TABLE 2.- DESIGN OF A FOUR-BAR PYRAMID

Areas of Bars

Load case	Bar	FSD		TFSD		AESOP		References 2 and 9	
		cm^2	in^2	cm^2	in^2	cm^2	in^2	cm^2	in^2
1	1	2.774	0.430	2.774	0.430	1.920	0.2976	2.774	0.430
	2	11.32	1.755	11.32	1.755	12.00	1.860	11.32	1.755
	3	8.116	1.258	8.116	1.258	7.142	1.107	8.116	1.258
	4	3.535	.548	3.535	.548	4.619	.716	3.535	.548
2	1	17.18	2.663	17.18	2.663	17.18	2.662	17.18	2.663
	2	14.82	2.298	14.82	2.298	14.81	2.295	14.82	2.298
	3	13.93	2.159	13.93	2.159	13.92	2.158	13.93	2.159
	4	.0	.0	.0	.0	.0	.0	.0	.0
3	1	0.3587	0.0556	0.3587	0.0556	0.3587	0.0556		
	2	2.254	.3494	2.254	.3493	2.254	.3493		
	3	.0	.0	.0	.0	.0	.0		
	4	1.664	.2580	1.664	.2580	1.664	.2580		
4	1	1.663	0.2578	1.663	0.2578	1.654	0.2563		
	2	1.193	.1850	1.193	.1850	1.200	.1860		
	3	1.482	.2297	1.482	.2297	1.471	.2280		
	4	.0	.0	.0	.0	.012	.0019		

Total Mass

Load case	FSD		TFSD		AESOP		References 2 and 9	
	kg	lbm	kg	lbm	kg	lbm	kg	lbm
1	29.83	65.76	29.83	65.76	29.80	65.71	29.83	65.76
2	52.25	115.2	52.25	115.2	52.25	115.2	52.29	115.3
3	5.008	11.04	5.008	11.04	5.008	11.04	5.008	11.04
4	5.008	11.04	5.008	11.04	5.008	11.04	5.008	11.04

TABLE 3.- ELEMENT CONNECTIONS AND GRID-POINT LOCATIONS FOR 25-BAR TRUSS

(a) Element connections

Element	Grid points		Element	Grid points	
1	1	2	14	3	10
2	1	4	15	6	7
3	2	3	16	4	9
4	1	5	17	5	8
5	2	6	18	4	7
6	2	4	19	3	8
7	2	5	20	5	10
8	1	3	21	6	9
9	1	6	22	6	10
10	3	6	23	3	7
11	4	5	24	5	9
12	3	4	25	4	8
13	5	6			

(b) Grid-point locations

Grid point	x		y		z	
	cm	in.	cm	in.	cm	in.
1	0	0	-63.5	-25	508	200
2	0	0	63.5	25	508	200
3	-95.25	-37.5	95.25	37.5	254	100
4	95.25	37.5	95.25	37.5	254	100
5	95.25	37.5	-95.25	-37.5	254	100
6	-95.25	-37.5	-95.25	-37.5	254	100
7	-254	-100	254	100	0	0
8	254	100	254	100	0	0
9	254	100	-254	-100	0	0
10	-254	-100	-254	-100	0	0

TABLE 4.- DESIGN OF A 25-BAR TRUSS

Areas of Bars

Bar	FSD		Taylor series FSD		TFSD		AESOP	
	cm^2	in^2	cm^2	in^2	cm^2	in^2	cm^2	in^2
1	0.0064	0.001	0.0064	0.001	0.0064	0.001	0.0064	0.001
2	1.504	.2331	1.502	.2328	1.518	.2353	.8671	.1344
3	1.314	.2037	1.316	.2040	1.326	.2056	.8361	.1296
4	.6419	.0995	.6406	.0993	.6542	.1014	.0903	.0140
5	1.279	.1983	1.282	.1987	1.293	.2005	.7297	.1131
6	1.722	.2669	1.719	.2665	1.710	.2650	2.194	.3401
7	1.217	.1886	1.214	.1881	1.202	.1863	1.763	.2732
8	1.323	.2050	1.325	.2054	1.308	.2028	1.953	.3027
9	.947	.1468	.9490	.1471	.9342	.1448	1.497	.2320
10	.0064	.001	.0064	.001	.0064	.001	.0064	.001
11	.0064	.001	.0064	.001	.0064	.001	.0064	.001
12	.6490	.1006	.6523	.1011	.6490	.1006	.6658	.1032
13	.3910	.0606	.3923	.0608	.3910	.0606	.3722	.0577
14	.5542	.0859	.5542	.0859	.5587	.0866	.3529	.0547
15	.3981	.0617	.3993	.0619	.3929	.0609	.6303	.0977
16	.7258	.1125	.7239	.1122	.7206	.1117	.9258	.1435
17	.2697	.0418	.2697	.0418	.2748	.0426	.0374	.0058
18	.0497	.0077	.0458	.0071	.0497	.0077	.0181	.0028
19	.0064	.001	.0064	.001	.0064	.001	.0064	.001
20	.0064	.001	.0064	.001	.0064	.001	.0084	.0013
21	.0064	.001	.0064	.001	.0064	.001	.0406	.0063
22	2.257	.3499	2.261	.3505	2.261	.3505	2.150	.3333
23	2.646	.4102	2.653	.4112	2.643	.4096	2.881	.4466
24	1.908	.2957	1.904	.2951	1.904	.2951	1.981	.3071
25	3.188	.4941	3.186	.4939	3.192	.4947	2.964	.4595

Total Mass

FSD		Taylor series FSD		TFSD		AESOP	
kg	lbm	kg	lbm	kg	lbm	kg	lbm
20.63	45.49	20.64	45.51	20.63	45.49	20.60	45.42

TABLE 5.- ELEMENT CONNECTIONS FOR WING

Element	Grid points		Element	Grid points		Element	Grid points		Element	Grid points	
1	1	3	35	18	25	69	10	16	103	21	17
2	3	7	36	19	26	70	14	21	104	22	18
3	7	11	37	23	32	71	15	22	105	15	11
4	11	17	38	24	33	72	16	22	106	16	12
5	17	23	39	25	34	73	20	28	107	10	7
6	23	31	40	2	5	74	21	29	108	6	3
7	8	12	41	5	9	75	22	30	109	1	2
8	12	18	42	9	14	76	27	36	110	6	4
9	18	24	43	14	20	77	28	37	111	5	3
10	24	32	44	20	27	78	29	38	112	10	8
11	19	25	45	27	35	79	27	31	113	9	7
12	25	33	46	10	15	80	20	23	114	16	13
13	26	34	47	15	21	81	14	17	115	15	12
14	3	4	48	21	28	82	9	11	116	14	11
15	7	8	49	28	36	83	5	7	117	22	19
16	11	12	50	22	29	84	2	3	118	21	18
17	12	13	51	29	37	85	28	32	119	20	17
18	17	18	52	30	38	86	21	24	120	30	26
19	18	19	53	5	6	87	15	18	121	29	25
20	23	24	54	9	10	88	10	12	122	28	24
21	24	25	55	14	15	89	29	33	123	27	23
22	25	26	56	15	16	90	22	25	124	38	34
23	31	32	57	20	21	91	30	34	125	37	33
24	32	33	58	21	22	92	22	26	126	36	32
25	33	34	59	27	28	93	16	19	127	35	31
26	1	4	60	28	29	94	10	13	128	3	10
27	3	8	61	29	30	95	6	8	129	7	15
28	4	8	62	35	36	96	2	4	130	11	21
29	7	12	63	36	37	97	36	31	131	12	22
30	8	13	64	37	38	98	37	32	132	17	28
31	11	18	65	2	6	99	38	33	133	18	29
32	12	19	66	5	10	100	28	23	134	23	36
33	13	19	67	6	10	101	29	24	135	24	37
34	17	24	68	9	15	102	30	25	136	25	38

TABLE 6.- GRID-POINT LOCATIONS FOR WING

Grid point	x		y		z	
	cm	in.	cm	in.	cm	in.
1	0	0	0	0	5.1	2
2	0	0	0	0	0	0
3	152	60	0	0	9.4	3.7
4	152	60	-41	-16	4.3	1.7
5	152	60	0	0	0	0
6	152	60	-41	-16	0	0
7	304	120	0	0	13.7	5.4
8	304	120	-81	-32	3.6	1.4
9	304	120	0	0	0	0
10	304	120	-81	-32	0	0
11	457	180	0	0	18	7.1
12	457	180	-81	-32	7.9	3.1
13	457	180	-122	-48	2.8	1.1
14	457	180	0	0	0	0
15	457	180	-81	-32	0	0
16	457	180	-122	-48	0	0
17	610	240	0	0	22.5	8.86
18	610	240	-81	-32	12.3	4.86
19	610	240	-163	-64	2.2	.86
20	610	240	0	0	0	0
21	610	240	-81	-32	0	0
22	610	240	-163	-64	0	0
23	762	300	0	0	26.9	10.6
24	762	300	-81	-32	16.8	6.6
25	762	300	-163	-64	6.6	2.6
26	762	300	-203	-80	1.45	.57
27	762	300	0	0	0	0
28	762	300	-81	-32	0	0
29	762	300	-163	-64	0	0
30	762	300	-203	-80	0	0
31	889	350	0	0	30.5	12.0
32	889	350	-81	-32	20.3	8.0
33	889	350	-163	-64	10.1	4.0
34	889	350	-203	-80	5.1	2.0
35	889	350	0	0	0	0
36	889	350	-81	-32	0	0
37	889	350	-163	-64	0	0
38	889	350	-203	-80	0	0

TABLE 7.- GRID-POINT LOADS AND TEMPERATURES FOR WING FOR LOAD CASE 1

Grid point	P_x		P_z		Temperature	
	N	lbf	N	lbf	K	°F
1					489	420
2			8 069	1814	489	420
3					478	400
4					489	420
5			12 104	2721	529	492
6			12 104	2721	489	420
7					478	400
8					489	420
9			16 138	3628	567	560
10			20 173	4535	489	420
11					472	390
12					478	400
13					489	420
14			16 138	3628	569	564
15			24 207	5442	556	540
16			16 138	3628	489	420
17					461	370
18					478	400
19					489	420
20			16 138	3628	564	556
21			24 207	5442	573	572
22			20 173	4535	489	420
23					444	340
24					467	380
25					478	400
26					489	420
27			16 138	3628	560	548
28			24 207	5442	569	564
29			24 207	5442	591	604
30			12 104	2721	489	420
31					422	300
32	183 480	41 250			456	360
33	183 480	41 250			478	400
34					489	420
35			8 069	1814	556	540
36	-183 480	-41 250	16 138	3628	564	556
37	-183 480	-41 250	16 138	3628	573	572
38			12 104	2721	489	420

TABLE 8.- DESIGN OF WING TRUSS AS DETERMINED BY FSD FOR LOAD CASE 1

[Total mass = 33.40 kg (73.64 lbm)]

Bar	Area		Bar	Area		Bar	Area		Bar	Area	
	cm^2	in^2		cm^2	in^2		cm^2	in^2		cm^2	in^2
1	0.0064	0.001	35	0.0064	0.001	69	0.0064	0.001	103	1.063	0.1648
2	↓	↓	36	.0064	.001	70	↓	↓	104	1.372	.2127
3	↓	↓	37	.0284	.0044	71	↓	↓	105	1.410	.2186
4	↓	↓	38	.0064	.001	72	↓	↓	106	.8664	.1343
5	↓	↓	39	↓	↓	73	↓	↓	107	1.237	.1918
6	↓	↓	40	↓	↓	74	↓	↓	108	.5490	.0851
7	↓	↓	41	↓	↓	75	↓	↓	109	.0064	.001
8	↓	↓	42	↓	↓	76	↓	↓	110	↓	↓
9	↓	↓	43	↓	↓	77	↓	↓	111	↓	↓
10	↓	↓	44	↓	↓	78	↓	↓	112	↓	↓
11	↓	↓	45	↓	↓	79	↓	↓	113	↓	↓
12	↓	↓	46	↓	↓	80	↓	↓	114	↓	↓
13	↓	↓	47	↓	↓	81	↓	↓	115	.0587	.0091
14	↓	↓	48	.0129	.0020	82	↓	↓	116	.0064	.001
15	↓	↓	49	1.870	.2899	83	↓	↓	117	.0064	.001
16	.8581	.1330	50	.0064	.001	84	↓	↓	118	.0368	.0057
17	.0064	.001	51	1.873	.2903	85	1.872	.2901	119	.0064	.001
18	1.368	.2121	52	.0064	.001	86	.0064	.001	120	.0064	.001
19	.0064	.001	53	.5348	.0829	87	.0064	.001	121	.0284	.0044
20	2.845	.4410	54	1.222	.1894	88	.0064	.001	122	.1677	.0260
21	.7677	.1190	55	2.230	.3456	89	1.872	.2902	123	.0064	.001
22	.0064	.001	56	.8510	.1319	90	.0064	.001	124	.0064	.001
23	.8252	.1279	57	2.385	.3697	91	↓	↓	125	.0877	.0136
24	.5006	.0776	58	1.357	.2104	92	↓	↓	126	.2581	.0400
25	.0064	.001	59	4.970	.7704	93	↓	↓	127	.0064	.001
26	↓	↓	60	2.823	.4375	94	↓	↓	128	↓	↓
27	↓	↓	61	.7606	.1179	95	↓	↓	129	↓	↓
28	↓	↓	62	.5542	.0859	96	↓	↓	130	↓	↓
29	↓	↓	63	.8013	.1242	97	.2664	.0413	131	↓	↓
30	↓	↓	64	.4942	.0766	98	.3161	.0490	132	↓	↓
31	↓	↓	65	.0064	.001	99	.5097	.0790	133	↓	↓
32	↓	↓	66	↓	↓	100	2.257	.3499	134	↓	↓
33	↓	↓	67	↓	↓	101	2.104	.3261	135	↓	↓
34	↓	↓	68	↓	↓	102	.7710	.1195	136	↓	↓

TABLE 9.- DESIGN OF WING TRUSS AS DETERMINED BY TFSD FOR LOAD CASE 1

[Total mass = 33.40 kg (73.64 lbm)]

Bar	Area		Bar	Area		Bar	Area		Bar	Area	
	cm^2	in^2		cm^2	in^2		cm^2	in^2		cm^2	in^2
1	0.0064	0.001	35	0.0064	0.001	69	0.0064	0.001	103	1.063	0.1648
2			36	.0064	.001	70			104	1.372	.2127
3			37	.0284	.0044	71			105	1.410	.2186
4			38	.0064	.001	72			106	.8664	.1343
5			39			73			107	1.237	.1918
6			40			74			108	.5490	.0851
7			41			75			109	.0064	.001
8			42			76			110		
9			43			77			111		
10			44			78			112		
11			45			79			113		
12			46			80			114		
13			47			81			115	.0587	.0091
14			48	.0129	.0020	82			116	.0064	.001
15			49	1.870	.2899	83			117	.0064	.001
16	.8581	.1330	50	.0064	.001	84			118	.0368	.0057
17	.0064	.001	51	1.873	.2903	85	1.872	.2901	119	.0064	.001
18	1.368	.2121	52	.0064	.001	86	.0064	.001	120	.0064	.001
19	.0064	.001	53	.5348	.0829	87	.0064	.001	121	.0284	.0044
20	2.845	.4410	54	1.222	.1894	88	.0064	.001	122	.1677	.0260
21	.7677	.1190	55	2.230	.3456	89	1.872	.2902	123	.0064	.001
22	.0064	.001	56	.8510	.1319	90	.0064	.001	124	.0064	.001
23	.8252	.1279	57	2.385	.3697	91			125	.0877	.0136
24	.5006	.0776	58	1.357	.2104	92			126	.2581	.0400
25	.0064	.001	59	4.970	.7704	93			127	.0064	.001
26			60	2.823	.4375	94			128		
27			61	.7606	.1179	95			129		
28			62	.5542	.0859	96			130		
29			63	.8013	.1242	97	.2664	.0413	131		
30			64	.4942	.0766	98	.3161	.0490	132		
31			65	.0064	.001	99	.5097	.0790	133		
32			66			100	2.257	.3499	134		
33			67			101	2.104	.3261	135		
34			68			102	.7710	.1195	136		

TABLE 10.- GRID-POINT LOADS AND TEMPERATURES FOR WING FOR LOAD CASE 2

Grid point	P_x		P_z		Temperature	
	N	lbf	N	lbf	K	°F
1					839	1050
2			8 069	1814	839	1050
3					811	1000
4					839	1050
5			12 104	2721	939	1230
6			12 104	2721	839	1050
7					811	1000
8					839	1050
9			16 138	3628	1033	1400
10			20 173	4535	839	1050
11					797	975
12					811	1000
13					839	1050
14			16 138	3628	1039	1410
15			24 207	5442	1006	1350
16			16 138	3628	839	1050
17					769	925
18					811	1000
19					839	1050
20			16 138	3628	1028	1390
21			24 207	5442	1050	1430
22			20 173	4535	839	1050
23					728	850
24					783	950
25					811	1000
26					839	1050
27			16 138	3628	1017	1370
28			24 207	5442	1039	1410
29			24 207	5442	1094	1510
30			12 104	2721	839	1050
31					672	750
32	183 480	41 250			756	900
33	183 480	41 250			811	1000
34					839	1050
35			8 069	1814	1006	1350
36	-183 480	-41 250	16 138	3628	1028	1390
37	-183 480	-41 250	16 138	3628	1050	1430
38			12 104	2721	839	1050

TABLE 11.- DESIGN OF WING TRUSS AS DETERMINED BY TFSD FOR LOAD CASE 2

[Total mass = 33.41 kg (73.65 lbm)]

Bar	Area		Bar	Area		Bar	Area		Bar	Area	
	cm^2	in^2		cm^2	in^2		cm^2	in^2		cm^2	in^2
1	0.0064	0.001	35	0.0064	0.001	69	0.0064	0.001	103	1.063	0.1648
2	↓	↓	36	.0064	.001	70	↓	↓	104	1.372	.2126
3	↓	↓	37	.0323	.0050	71	↓	↓	105	1.410	.2186
4	↓	↓	38	.0064	.001	72	↓	↓	106	.8664	.1343
5	↓	↓	39	↓	↓	73	↓	↓	107	1.237	.1917
6	↓	↓	40	↓	↓	74	↓	↓	108	.5490	.0851
7	↓	↓	41	↓	↓	75	↓	↓	109	.0064	.001
8	↓	↓	42	↓	↓	76	↓	↓	110	↓	↓
9	↓	↓	43	↓	↓	77	↓	↓	111	↓	↓
10	↓	↓	44	↓	↓	78	↓	↓	112	↓	↓
11	↓	↓	45	↓	↓	79	↓	↓	113	↓	↓
12	↓	↓	46	↓	↓	80	↓	↓	114	↓	↓
13	↓	↓	47	↓	↓	81	↓	↓	115	.0587	.0091
14	↓	↓	48	.0155	.0024	82	↓	↓	116	.0064	.001
15	↓	↓	49	1.870	.2899	83	↓	↓	117	.0064	.001
16	.8581	.1330	50	.0064	.001	84	↓	↓	118	.0368	.0057
17	.0064	.001	51	1.874	.2904	85	1.869	.2897	119	.0064	.001
18	1.368	.2121	52	.0064	.001	86	.0064	.001	120	.0064	.001
19	.0064	.001	53	.5348	.0829	87	.0064	.001	121	.0284	.0044
20	2.844	.4409	54	1.222	.1894	88	.0064	.001	122	.1677	.0260
21	.7677	.1190	55	2.230	.3456	89	1.872	.2902	123	.0064	.001
22	.0064	.001	56	.8510	.1319	90	.0064	.001	124	.0064	.001
23	.8271	.1282	57	2.386	.3698	91	↓	↓	125	.0877	.0136
24	.5006	.0776	58	1.357	.2104	92	↓	↓	126	.2581	.0400
25	.0064	.001	59	4.969	.7702	93	↓	↓	127	.0064	.001
26	↓	↓	60	2.823	.4375	94	↓	↓	128	↓	↓
27	↓	↓	61	.7606	.1179	95	↓	↓	129	↓	↓
28	↓	↓	62	.5548	.0860	96	↓	↓	130	↓	↓
29	↓	↓	63	.8013	.1242	97	.2658	.0412	131	↓	↓
30	↓	↓	64	.4942	.0766	98	.3161	.0490	132	↓	↓
31	↓	↓	65	.0064	.001	99	.5097	.0790	133	↓	↓
32	↓	↓	66	↓	↓	100	2.256	.3497	134	↓	↓
33	↓	↓	67	↓	↓	101	2.104	.3261	135	↓	↓
34	↓	↓	68	↓	↓	102	.7710	.1195	136	↓	↓

TABLE 12.- GRID-POINT LOADS AND TEMPERATURES FOR WING

FOR LOAD CASE 3

Grid point	P_X		P_Z		Temperature	
	N	lbf	N	lbf	K	°F
1					547	525
2			8 069	1814	547	525
3					533	500
4					547	525
5			12 104	2721	597	615
6			12 104	2721	547	525
7					533	500
8					547	525
9			16 138	3628	644	700
10			20 173	4535	547	525
11					526	488
12					533	500
13					547	525
14			16 138	3628	647	705
15			24 207	5442	630	675
16			16 138	3628	547	525
17					513	463
18					533	500
19					547	525
20			16 138	3628	642	695
21			24 207	5442	653	715
22			20 173	4535	547	525
23					492	425
24					519	475
25					533	500
26					547	525
27			16 138	3628	636	685
28			24 207	5442	647	705
29			24 207	5442	675	755
30			12 104	2721	547	525
31					464	375
32	183 480	41 250			506	450
33	183 480	41 250			533	500
34					547	525
35			8 069	1814	630	675
36	-183 480	-41 250	16 138	3628	642	695
37	-183 480	-41 250	16 138	3628	653	715
38			12 104	2721	547	525

TABLE 13.- DESIGN OF WING TRUSS AS DETERMINED BY TFSD FOR LOAD CASE 3

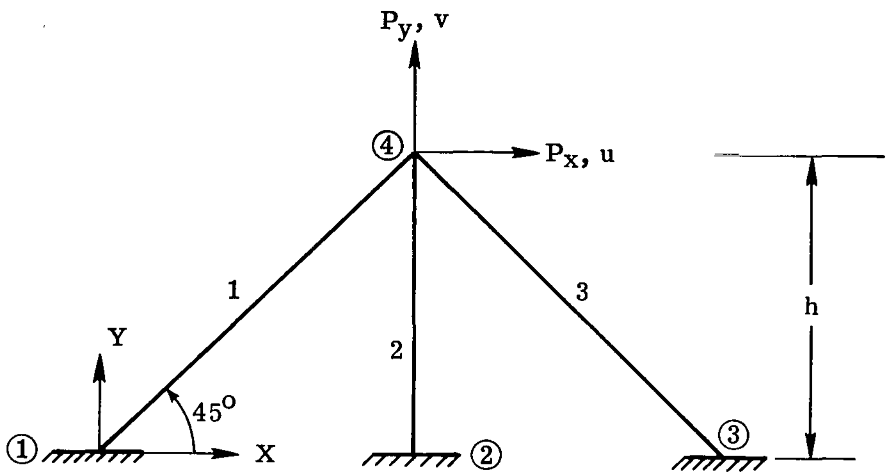
[Total mass = 33.40 kg (73.64 lbm)]

Bar	Area		Bar	Area		Bar	Area		Bar	Area	
	cm^2	in^2		cm^2	in^2		cm^2	in^2		cm^2	in^2
1	0.0064	0.001	35	0.0064	0.001	69	0.0064	0.001	103	1.063	0.1648
2	↓	↓	36	.0064	.001	70	↓	↓	104	1.372	.2127
3			37	.0342	.0053	71			105	1.410	.2186
4			38	.0064	.001	72			106	.8664	.1343
5			39	↓	↓	73			107	1.237	.1918
6			40			74			108	.5490	.0851
7			41			75			109	.0064	.001
8			42			76			110	↓	↓
9			43			77			111		
10			44			78			112		
11			45			79			113		
12			46			80			114		
13			47	.0084	.0013	81			115	.0587	.0091
14			48	.0129	.0020	82			116	.0064	.001
15			49	1.837	.2848	83			117	.0064	.001
16	.0857	.1329	50	.0064	.001	84			118	.0368	.0057
17	.0064	.001	51	1.872	.2902	85	1.864	.2889	119	.0064	.001
18	1.368	.2120	52	.0064	.001	86	.0064	.001	120	.0064	.001
19	.0064	.001	53	.5348	.0829	87	.0064	.001	121	.0284	.0044
20	2.844	.4409	54	1.222	.1894	88	.0064	.001	122	.1677	.0260
21	.7677	.1190	55	2.230	.3457	89	1.873	.2903	123	.0064	.001
22	.0064	.001	56	.8510	.1319	90	.0064	.001	124	.0064	.001
23	.8290	.1285	57	2.387	.3700	91	↓	↓	125	.0877	.0136
24	.5006	.0776	58	1.357	.2104	92			126	.2574	.0399
25	.0064	.001	59	4.965	.7696	93			127	.0064	.001
26	↓	↓	60	2.822	.4374	94			128	↓	↓
27			61	.7606	.1179	95			129		
28			62	.5574	.0864	96			130		
29			63	.8019	.1243	97	.2369	.0409	131		
30			64	.4942	.0766	98	.3155	.0489	132		
31			65	.0064	.001	99	.5097	.0790	133		
32			66	↓	↓	100	2.254	.3493	134		
33			67			101	2.104	.3261	135		
34			68			102	.7710	.1195	136		

TABLE 14.- DESIGN OF WING TRUSS AS DETERMINED BY FSD FOR LOAD CASE 3

[Total mass = 33.40 kg (73.64 lbm)]

Bar	Area		Bar	Area		Bar	Area		Bar	Area	
	cm^2	in^2		cm^2	in^2		cm^2	in^2		cm^2	in^2
1	0.0064	0.001	35	0.0064	0.001	69	0.0064	0.001	103	1.063	0.1647
2	↓	↓	36	.0064	.001	70	↓	↓	104	1.372	.2126
3	↓	↓	37	.0484	.0075	71	↓	↓	105	1.410	.2185
4	↓	↓	38	.0064	.001	72	↓	↓	106	.8664	.1343
5	↓	↓	39	↓	↓	73	↓	↓	107	1.237	.1917
6	↓	↓	40	↓	↓	74	↓	↓	108	.5490	.0851
7	↓	↓	41	↓	↓	75	↓	↓	109	.0064	.001
8	↓	↓	42	↓	↓	76	↓	↓	110	↓	↓
9	↓	↓	43	↓	↓	77	↓	↓	111	↓	↓
10	↓	↓	44	↓	↓	78	↓	↓	112	↓	↓
11	↓	↓	45	↓	↓	79	↓	↓	113	↓	↓
12	↓	↓	46	↓	↓	80	↓	↓	114	↓	↓
13	↓	↓	47	.0142	.0022	81	↓	↓	115	.0587	.0091
14	↓	↓	48	.0264	.0041	82	↓	↓	116	.0064	.001
15	↓	↓	49	1.870	.2898	83	↓	↓	117	.0064	.001
16	.8581	.1330	50	.0064	.001	84	↓	↓	118	.0368	.0057
17	.0064	.001	51	1.874	.2904	85	1.855	.2876	119	.0064	.001
18	1.367	.2120	52	.0064	.001	86	.0064	.001	120	.0064	.001
19	.0064	.001	53	.5348	.0829	87	.0064	.001	121	.0284	.0044
20	2.844	.4409	54	1.223	.1895	88	.0064	.001	122	.1677	.0260
21	.7677	.1190	55	2.232	.3459	89	1.872	.2902	123	.0064	.001
22	.0064	.001	56	.8510	.1319	90	.0064	.001	124	.0064	.001
23	.8355	.1295	57	2.386	.3699	91	↓	↓	125	.0877	.0136
24	.5006	.0776	58	1.357	.2104	92	↓	↓	126	.2561	.0397
25	.0064	.001	59	4.963	.7693	93	↓	↓	127	.0064	.001
26	↓	↓	60	2.822	.4374	94	↓	↓	128	↓	↓
27	↓	↓	61	.7606	.1179	95	↓	↓	129	↓	↓
28	↓	↓	62	.5594	.0867	96	↓	↓	130	↓	↓
29	↓	↓	63	.8006	.1241	97	.2606	.0404	131	↓	↓
30	↓	↓	64	.4942	.0766	98	.3161	.0490	132	↓	↓
31	↓	↓	65	.0064	.001	99	.5097	.0790	133	↓	↓
32	↓	↓	66	↓	↓	100	2.249	.3486	134	↓	↓
33	↓	↓	67	↓	↓	101	2.104	.3261	135	↓	↓
34	↓	↓	68	↓	↓	102	.7710	.1195	136	↓	↓



Load case	P_x		P_y		$(\alpha T)_1$	$(\alpha T)_2$	$(\alpha T)_3$
	N	lbf	N	lbf			
1	4.448×10^5	10^5	0	0	6.25×10^{-4}	1.25×10^{-3}	1.875×10^{-3}
2	-424620	-95459	424620	95459	1.875×10^{-3}	1.25×10^{-3}	6.25×10^{-4}
3	364380	81915	255140	57358	0	0	0

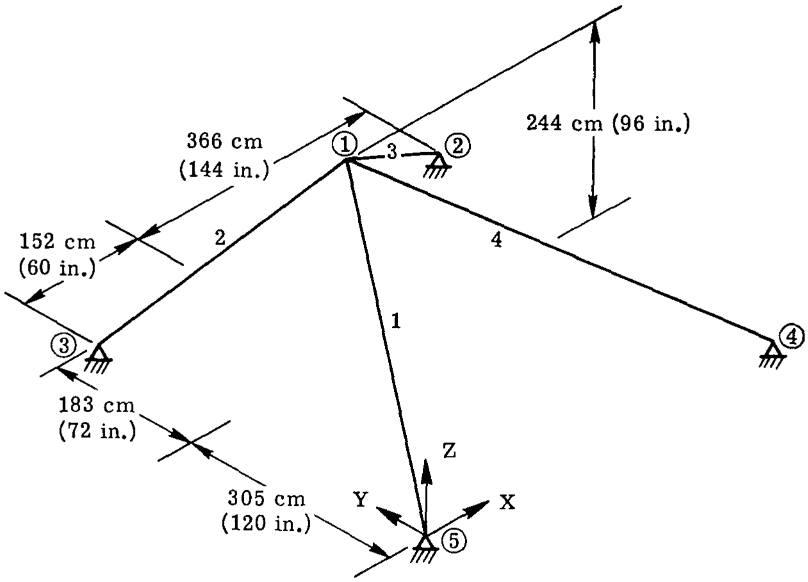
$E = 71.7$ GPa $(10.4 \times 10^6$ psi)

$\rho = 2800$ kg/m^3 (0.101 lbm/in^3)

$$\sigma_a = \begin{cases} 0.492 \text{ GPa } (71\ 500 \text{ psi}) \\ -0.350 \text{ GPa } (-50\ 700 \text{ psi}) \end{cases}$$

h = 50.8 cm (20 in.)

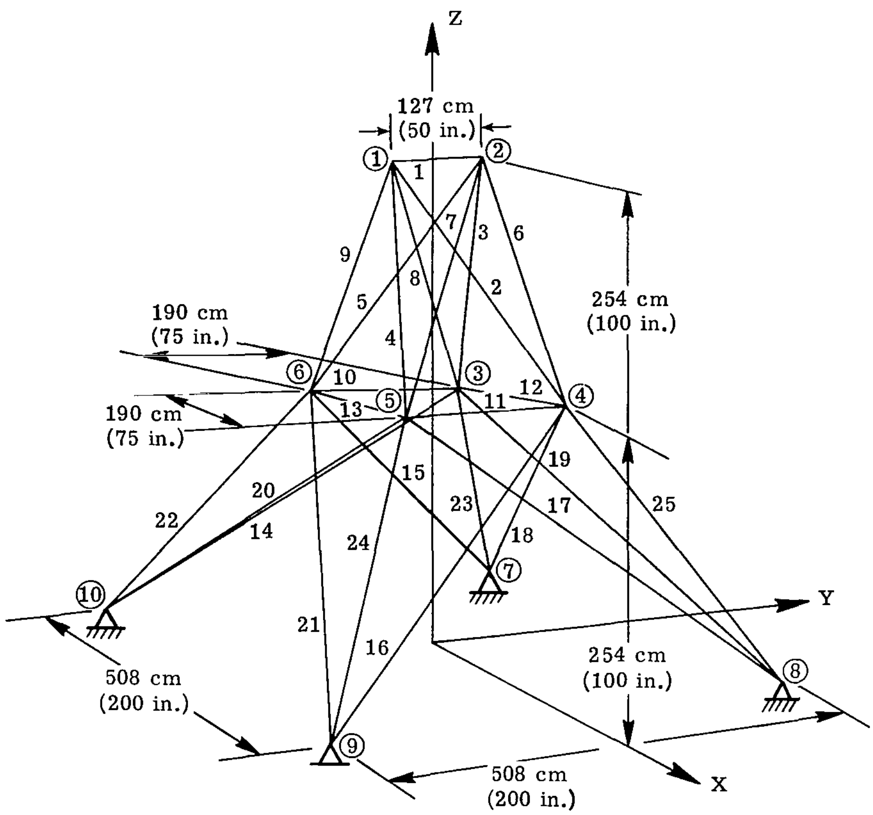
Figure 1.- Three-bar truss.



		Case 1	Case 2	Case 3	Case 4
P_x	N (lbf)	4.448×10^4 (1×10^4)	1.779×10^5 (4×10^4)		
P_y	N (lbf)	8.896×10^4 (2×10^4)	4.448×10^5 (1×10^5)		
P_z	N (lbf)	-2.669×10^5 (-6×10^4)	-1.334×10^5 (-3×10^4)	4.448×10^4 (1×10^4)	-4.448×10^4 (-1×10^4)
T_1	K (°F)				
T_2	K (°F)			372 (200)	372 (200)
T_3	K (°F)			372 (200)	372 (200)
T_4	K (°F)			311 (100)	311 (100)
T_5	K (°F)			311 (100)	311 (100)

$E = 68.9$ GPa (1×10^7 psi)
$\rho = 2770$ kg/m^3 (0.100 lbm/in^3)
$\sigma_a = \pm 0.17$ GPa ($\pm 25\ 000$ psi)
$\alpha = 22.5 \times 10^{-6}$/K ($12.5 \times 10^{-6}$/°F)

Figure 2.- Four-bar pyramid. All forces applied at point 1.



Grid point	Temperature		P_x		P_y		P_z	
	K	°F	N	lbf	N	lbf	N	lbf
1	350	170	4448	1000	44 480	10 000	-22 240	-5000
2	350	170	4448	1000	44 480	10 000	-22 240	-5000
3	311	100	2224	500				
4	311	100						
5	311	100						
6	311	100	2224	500				
7	275	35						
8	275	35						
9	275	35						
10	275	35						

$E = 68.9$ GPa (1×10^7 psi)
$\rho = 2800$ kg/m^3 (0.101 lbm/in^3)
$\sigma_a = \pm 0.276$ GPa ($\pm 40\,000$ psi)
$\alpha = 23.0 \times 10^{-6}$/K ($12.8 \times 10^{-6}$/°F)

Figure 3.- A 25-bar transmission tower.

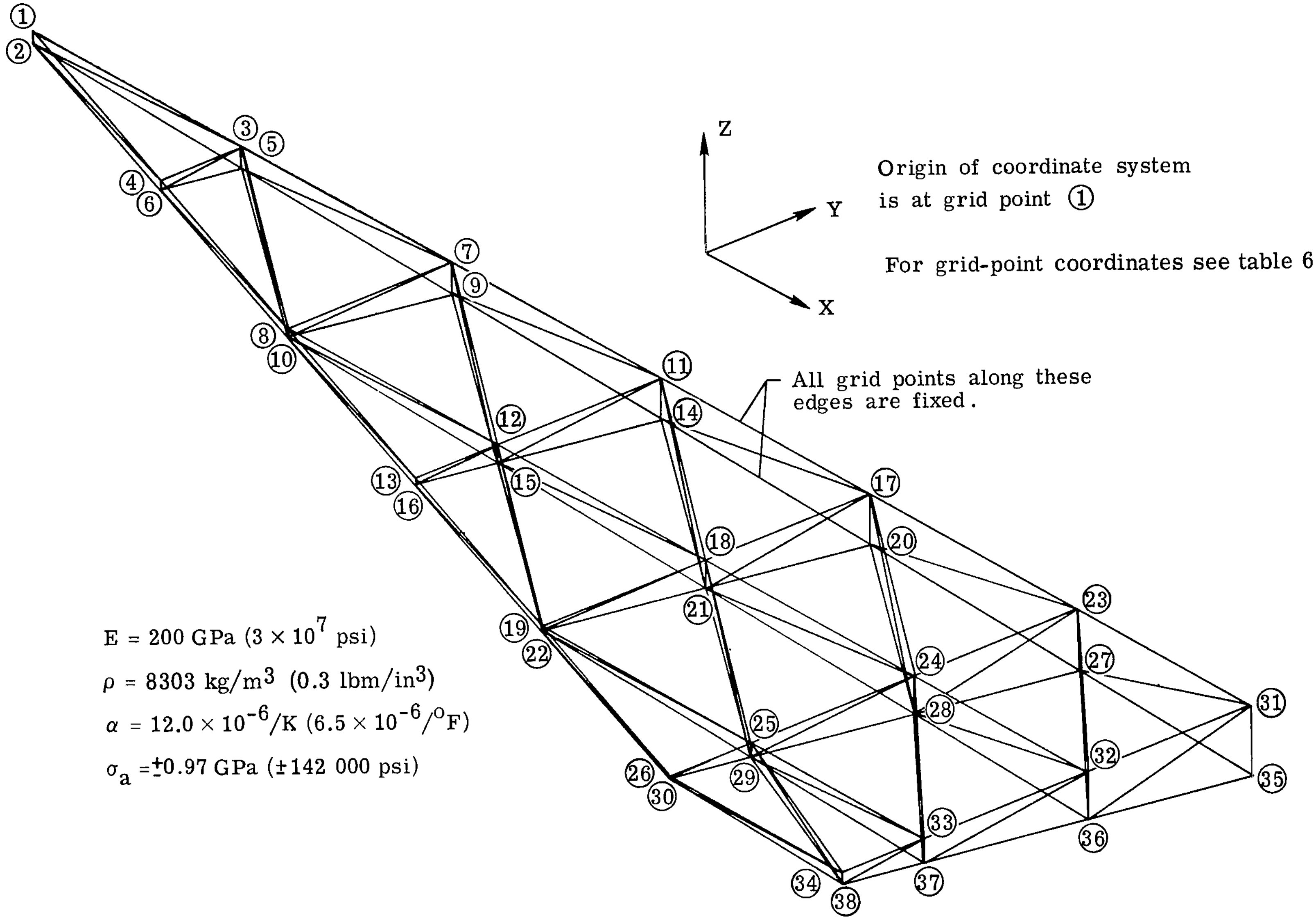


Figure 4.- A 136-bar model of hypersonic wing.

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